

# 4<sup>th</sup> Alpen-Adria-Workshop on Optimization

Alpen-Adria-Universität Klagenfurt, Room K.0.01\*

November 4–6, 2016, <http://aawo2016.aau.at/>

## Friday, November 4, 2016

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8:40	–	9:00	registration
9:00	–	9:30	<i>Kurt Anstreicher</i> : Maximum Entropy Sampling and the Boolean Quadric Polytope
9:35	–	10:05	<i>Miguel Anjos</i> : Computational Study of Some Valid Inequalities for $k$ -Way Graph Partitioning
10:10	–	10:30	break
10:30	–	11:00	<i>Marianna De Santis</i> : Dual Bounds for (Mixed) Integer Quadratic Programming Problems
11:05	–	11:25	<i>Eranda Dragoti-Çela</i> : A New Solvable Case of the QAP and Lower Bounds
11:30	–	11:50	<i>Anna Jellen</i> : The Travelling Salesperson Problem with Forbidden Neighborhoods on Regular 2D and 3D Grids
11:55	–	12:15	<i>Ulrich Pferschy</i> : Fair Allocation: Assigning Items and Budgets
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lunch break (including after-lunch nap) <i>Franz likes taking a nap after lunch.</i>			
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14:15			Festkolloquium, see attached invitation

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## Saturday, November 5, 2016

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9:00	–	9:30	<i>Monique Laurent</i> : On the Cone of Completely Positive Semidefinite Matrices
9:35	–	10:05	<i>Sam Burer</i> : A Copositive Approach for Two-Stage Adjustable Robust Optimization with Uncertain Right-Hand Sides
10:10	–	10:30	break
10:30	–	11:00	<i>Henry Wolkowicz</i> : Low-Rank Matrix Completion (LRMC) using Nuclear Norm (NN) with Facial Reduction (FR)
11:05	–	11:25	<i>tba</i> : Satalia - Solving Hard Problems
11:30	–	11:50	<i>Igor Đukanović</i> : An SDP Approach to Graph Coloring
11:55	–	12:15	<i>Bettina Klinz</i> : On an Untypical Type of Assignment Problems
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lunch break (including after-lunch nap)			
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14:00	–	14:30	<i>Tamás Terlaky</i> : A Polynomial Column-wise Rescaling von Neumann Algorithm
14:35	–	15:05	<i>Jon Lee</i> : Volumetric Tuning of sBB for Global Optimization of Factorable Formulations
15:10	–	15:20	break
15:20	–	15:40	<i>Elisabeth Gaar</i> : One of Franz' Most Recent Toys: Exact Subgraph Constraints
15:45	–	16:05	<i>Janez Povh</i> : A New Conic LP Approximation Hierarchy for Polynomial Conic Optimization
16:10	–	16:30	<i>Philipp Hungerländer</i> : A Feasible Active Set Method for Asymmetric Complementarity Problems
16:35	–	16:55	<i>Renata Sotirov</i> : On Solving the Quadratic Shortest Path Problem

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## Sunday, November 6, 2016

*Short and easy hike.* Details will be announced during the workshop. We will be back such that the flight to Vienna at 6.30 p.m. can be easily reached.

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\*Go to the building marked “Sparkasse” and “Cafe Como”. Room K.0.01 is on the ground floor of that building.

## Abstracts

*Anna Jellen:* The Travelling Salesperson Problem with Forbidden Neighborhoods on Regular 2D and 3D Grids

We suggest and examine an extension of the Travelling Salesperson Problem (TSP) motivated by an application in mechanical engineering. The TSP with forbidden neighborhoods (TSPFN) with radius  $r$  is asking for a shortest Hamiltonian cycle of a given graph  $G$ , where vertices traversed successively have a distance larger than  $r$ .

The TSPFN is motivated by an application in mechanical engineering, more precisely in laser beam melting. This technology is used for building complex workpieces in several layers, similar to 3D printing. For each layer new material has to be heated up at several points. The question is now how to choose the order of the points to be treated in each layer such that internal stresses are low. Furthermore, one is interested in low cycle times of the workpieces. One idea is to look for short paths between the points or more precisely between the segments in each layer that do not connect segments that are too close so that the heat quantity in each region is not too high in short periods. In particular in the instances resulting from this application the layers are rectangular non-regular grids.

In this work we start with the consideration of regular grids, i.e., adjacent vertices in the same row or column all have the same distance from each other. First we suggest a linear integer programming formulation of TSPFN. Then we examine TSPFN with  $r = 0$ ,  $r = 1$  and  $r = \sqrt{2}$ . We determine the length and structure of optimal solutions and show that these problems can be solved in linear time. After discussing optimal TSPFN tours in the plane we briefly consider the three dimensional case and determine optimal TSPFN tours for  $r = 0$  and  $r = 1$  on regular 3D grids. This is joint work with Anja Fischer and Philipp Hungerländer.

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*Elisabeth Gaar:* One of Franz' Most Recent Toys: Exact Subgraph Constraints

Franz is playing with exact subgraph constraints for quite some time now. We will have fun using them for the stable set problem. This is a NP-hard problem, where one wants to find the biggest set of vertices in a graph such that no two vertices are adjacent.

An upper bound on the stability number of a graph is the Lovász theta function, which can be computed in polynomial time as semidefinite program (SDP). One possibility to further improve this upper bound is to include so called exact subgraph constraints into the SDP. For a certain subgraph the exact subgraph constraint ensures, that the submatrix of the calculation of the Lovász theta function corresponding to the subgraph is contained in the convex hull of all stable set matrices of the subgraph.

Recently Franz, Malwina Duda and I are trying to solve the obtained SDP by building the Lagrangian dual with respect to the exact subgraph constraints and applying the bundle method with easy components to the resulting SDP. On our way we will encounter a very nicely structured QP.

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*Eranda Dragoti-Çela:* A New Solvable Case of the QAP and Lower Bounds

The quadratic assignment problem (QAP), a classical problem in combinatorial optimization, is very hard to deal with, both from the theoretical and from the practical point of view. Polynomially solvable special cases (PSSC) of the problem arise by imposing certain structural and generally quite restrictive properties on its input matrices.

We consider the usage of PSSCs to obtain lower bounds for the general problem. More precisely, we approximate the input matrices of a general instance of the QAP by matrices with particular structural properties which make the QAP polynomially solvable, such that the optimal objective function value of the involved PSSC is a lower bound for the original problem. This approximation is done in a stepwise

approach, where in every step certain particularly structured matrices are subtracted from the current permuted coefficient matrices of the QAP so as to obtain an increasing sequence of bounds.

We consider a new PSSC of the QAP which involves coefficient matrices with a special diagonal structure. By combining this new PSSC with PSSCs known in the literature we obtain a new and larger class of polynomially solvable QAPs which seems to be amenable to the computation of lower bounds. We show that the recognition problem for the new PSSC can be solved in polynomial time.

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*Henry Wolkowicz: Low-Rank Matrix Completion (LRMC) using Nuclear Norm (NN) with Facial Reduction (FR)*

Minimization of the NN is often used as a surrogate, convex relaxation, for solving LRMC problems. The minimum NN problem can be solved as a trace minimization semidefinite program (SDP). The SDP and its dual are regular in the sense that they both satisfy strict feasibility. FR has been successful in regularizing many problems where strict feasibility fails, e.g., SDP relaxations of discrete optimization problems such as QAP, GP, as well as sensor network localization. Here we take advantage of the structure at optimality for the NN minimization and show that even though strict feasibility holds, the FR framework can be successfully applied to obtain a proper face that contains the optimal set. This can dramatically reduce the size of the final NN problem while guaranteeing a low-rank solution. We include numerical tests for both exact and noisy cases. This is joint work with Shimeng Huang.

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*Igor Đukanović: An SDP Approach to Graph Coloring*

Graph coloring has a canonic representation by binary matrices leading to a copositive representation of (fractional) chromatic number. Its SDP relaxation can be efficiently computed on vertex transitive graphs, while impressively improving Lovasz bound.

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*Janez Povh: A New Conic LP Approximation Hierarchy for Polynomial Conic Optimization*

In this paper we consider polynomial conic optimization problems, where the feasible set is defined by constraints in the form of given polynomial vectors belonging to given nonempty closed convex cones, and we assume that all the feasible solutions are nonnegative.

After translation, this family of problems captures in particular compact polynomial optimization problems, compact polynomial semidefinite problems and compact polynomial second order cone optimization problems.

We propose a general hierarchy of conic linear programming relaxations which is under some classical assumptions monotonic and converges to the optimal value of the original problem. The members of this hierarchy provide strong bounds for the optimum value and are in special cases from previous paragraph much easier to compute compared to classical SOS and moment approximations.

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*Jon Lee: Volumetric Tuning of sBB for Global Optimization of Factorable Formulations*

Using volume calculations, I will demonstrate how we can get mathematical guidance for convexification strategies and for branching-point selection, in the context of the spatial branch-and-bound algorithm for factorable global-optimization formulations. We carry this out in considerable detail for the basic function of multiplying three terms. This is joint work with Emily Speakman.

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*Kurt Anstreicher: Maximum Entropy Sampling and the Boolean Quadric Polytope.*

We consider a bound for the maximum-entropy sampling problem (MESP) that is based on solving a max-det problem over a relaxation of the Boolean Quadric Polytope (BQP). This approach to MESP was first suggested by Christoph Helmberg over 15 years ago, but has apparently never been further elaborated or computationally investigated. We find that the use of a relaxation of BQP that imposes semidefiniteness and a small number of equality constraints gives outstanding bounds on many benchmark instances. These bounds can be further tightened by imposing additional inequality constraints that are valid for the BQP. On some problems the bounds are sufficiently tight so that variables can be set to 0/1 values using variable-fixing logic for semidefinite programming.

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*Marianna De Santis: Dual Bounds for (Mixed) Integer Quadratic Programming Problems*

We will talk about branch-and-bound algorithms that generalize the approach for unconstrained convex quadratic integer programming proposed by Buchheim, Caprara and Lodi [Math, Program., 135 (2012), pp. 369-395] to the presence of linear constraints and nonconvex objective function. The main feature of the latter approach consists of a sophisticated preprocessing phase, leading to a fast enumeration on the branch-and-bound nodes. The focus of the talk will be on how lower bounds are computed. Experimental results on randomly generated instances show that the approach significantly outperforms the MIQP solver of CPLEX 12.6 for instances with a small number of constraints.

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*Miguel Anjos: Computational Study of Some Valid Inequalities for  $k$ -Way Graph Partitioning*

We consider the maximum  $k$ -cut problem that consists in partitioning the vertex set of a graph into  $k$  subsets such that the sum of the weights of edges joining vertices in different subsets is maximized. We focus on identifying effective classes of inequalities to tighten the semidefinite programming relaxation. We carry out an experimental study of four classes of inequalities from the literature: clique, general clique, wheel and bicycle wheel. We considered multiple combinations of these classes and tested them on both dense and sparse instances for different values of  $k$ . Our computational results suggest that the bicycle wheel and wheel are the strongest inequalities for  $k = 3$ , and that for greater  $k$ , the wheel inequalities are the strongest by far. Furthermore, we observe an improvement in the performance for all choices of  $k$  when both bicycle wheel and wheel are used, at the cost of substantially increased CPU time on average when compared with using only one of them. This is joint work with Vilmar Jefte Rodrigues de Sousa and Sebastien Le Digabel.

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*Monique Laurent: On the Cone of Completely Positive Semidefinite Matrices*

We will discuss properties of the completely positive semidefinite cone, a matrix cone which consists of all matrices that admit a Gram representation by positive semidefinite matrices (of any size). This matrix cone can be used to model quantum analogues of classical graph parameters like graph coloring and it can be seen as an extension of the classical completely positive cone.

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*Philipp Hungerländer: A Feasible Active Set Method for Asymmetric Complementarity Problems*

A primal feasible active set method is presented which extends the globally convergent semismooth Newton method for strictly convex quadratic problems with simple bounds proposed by [P. Hungerländer and F. Rendl. A feasible active set method for strictly convex problems with simple bounds. *SIAM Journal on Optimization*, 25(3):1633–1659, 2015] to linear complementarity problems with P-matrices. Based on a guess of the active set, a primal-dual pair  $(x, u)$  is computed that satisfies stationarity and the complementary condition. If  $x$  is not feasible, the variables connected to the infeasibilities are added to the active set and a new primal-dual pair  $(x, u)$  is computed. This process is iterated until a primal feasible solution is generated. Then a new active set is determined based on the feasibility information of the dual variable  $u$ . We prove that the algorithm stops after a finite number of steps with an optimal solution if the linear complementarity problem has a unique solution. Computational experience indicates that this approach performs very well in practice.

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*Renata Sotirov: On Solving the Quadratic Shortest Path Problem*

We study the quadratic shortest path problem (QSPP) which is the shortest path problem with the objective of minimizing the sum of interaction costs over each pair of arcs on the path. This problem is known to be NP-hard. In this talk we consider linearizable QSPP instances whose optimal solution can be found by solving the corresponding instance of the shortest path problem. We also present an algorithm that verifies whether a QSPP instance on the directed grid graph is linearizable, and if it is linearizable the algorithm returns the corresponding linearization vector.

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*Sam Burer: A Copositive Approach for Two-Stage Adjustable Robust Optimization with Uncertain Right-Hand Sides*

We study two-stage adjustable robust linear programming in which the right-hand sides are uncertain and belong to a convex, compact uncertainty set. This problem is NP-hard, and the affine policy is a popular, tractable approximation. We prove that under standard and simple conditions, the two-stage problem can be reformulated as a copositive optimization problem, which in turn leads to a class of tractable, semidefinite-based approximations that are at least as strong as the affine policy. We investigate several examples from the literature demonstrating that our tractable approximations significantly improve the affine policy. In particular, our approach solves exactly in polynomial time a class of instances of increasing size for which the affine policy admits an arbitrarily large gap. This is joint work with Guanglin Xu.

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*Tamás Terlaky: A Polynomial Column-wise Rescaling von Neumann Algorithm*

Recently Chubanov proposed a method which solves homogeneous linear equality systems with positive variables in polynomial time. Chubanov's method can be considered as a column-wise rescaling procedure. We adapt Chubanov's method to the von Neumann problem, and so we design a polynomial time column-wise rescaling von Neumann algorithm. This algorithm is the first variant of the von Neumann algorithm with polynomial complexity. This is joint work with Dan Li and Kees Roos.

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*tba: Satalia - Solving Hard Problems*

Satalia is a British company specialized in solving hard combinatorial optimization problems, in particular routing, planning, scheduling and verification problems. Satalia has a research collaboration with the University of Klagenfurt, and many industry clients. In this talk we give an overview of ongoing research projects with industry, their goals and results, as well as ongoing research on developing a high-level portfolio-style SolveEngine that aggregates thousands of solving approaches to tackle hard real-world problems.

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*Ulrich Pferschy: Fair Allocation: Assigning Items and Budgets*

Franz is well-known for being a very fair person. Hence, we study fairness as a measure for allocating items to agents and compare the outcome to a system optimal solution from a worst-case perspective. Remembering the administrative duties Franz had to face, we also perform a case-study in a budget allocation setting and characterize the profit-fairness trade-off.

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## List of Participants

Angelika Wiegele	Alpen-Adria-Universität Klagenfurt
Anna Jellen	Alpen-Adria-Universität Klagenfurt
Bettina Klinz	TU Graz
Christian Truden	Alpen-Adria-Universität Klagenfurt
Christoph Helmberg	TU Chemnitz
Elisabeth Gaar	Alpen-Adria-Universität Klagenfurt
Eranda Dragoti-Çela	TU Graz
Francesco Silvestri	Universität Heidelberg
Franz Rendl	Alpen-Adria-Universität Klagenfurt
Friedwin Sturm	Montanuniversität Leoben
Gerald Gruber	FH Kärnten
Gerhard Reinelt	Universität Heidelberg
Giovanni Rinaldi	IASI Rome
Henry Wolkowicz	University of Waterloo
Igor Đukanović	Univerza v Mariboru
Janez Povh	Univerza v Ljubljani
Joachim Schauer	Universität Graz
Jon Lee	University of Michigan
Kerstin Maier	Alpen-Adria-Universität Klagenfurt
Kurt Anstreicher	University of Iowa
Malwina Duda	Alpen-Adria-Universität Klagenfurt
Marianna De Santis	Università di Padova
Markus Aschinger	Alpen-Adria-Universität Klagenfurt
Michael Jünger	Universität zu Köln
Miguel F. Anjos	Polytechnique Montréal
Monique Laurent	CWI Amsterdam
Philipp Hungerländer	Alpen-Adria-Universität Klagenfurt
Rainer Burkard	TU Graz
Renata Sotirov	Tilburg University
Sam Burer	University of Iowa
Tamás Terlaky	Lehigh University
Ulrich Pfersch	Universität Graz

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